On the Comparative Performance Analysis of PAPR Reduction Techniques in OFDM System

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Abstract—Selected Mapping (SLM) and Partial Transmit Sequence (PTS) are two very well-known Peak-to-average Power Ratio (PAPR) reduction techniques for Orthogonal Frequency Division Multiplexing (OFDM) systems. Both these schemes show good PAPR reduction capabilities. However, for any PAPR reduction technique, the nature of spectral occupancy and the associated computational complexity also need to be taken into account when the overall performance is considered. In this paper, our goal is to perform a comparative performance analysis of SLM and PTS techniques by considering these three parameters, i.e. PAPR reduction, computational overhead and spectral compactness. For this, we at first look for the optimum values in terms of number of sequences in SLM and no. of sub-blocks in PTS. And then based on this finding, we perform spectral analysis. Our finding shows that, PTS outperforms SLM when compared on the parameters mentioned above.

IndexTerms—OFDM, PAPR, CCDF, PTS, SLM, Spectral Occupancy.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation technique that is capable of achieving high data rate, good spectral efficiency, robustness in frequency selective fading channels and immunity to intersymbol interference [1]. This technique is already being used in Digital Audio and video Broadcasting (DAB and DVB), LAN (IEEE 802.11x) and WiMAX and so forth [1][2].

There are certain challenges in implementing OFDM also. In particular, the issue of high Peak to Average Power Ratio (PAPR) of the transmitted signal [3] needs special attention. High PAPR may drive transmitter high power amplifier (HPA) into its saturation region resulting in nonlinear amplification. This non-linear amplification eventually results in severe bit error rate (BER) degradation in the receiver. One solution to this problem is to operate HPAs with high-back offs to compensate for large peaks present in the signal. But this yields in very poor power efficiency. Since, wireless handheld terminals are by nature power constrained, it is imperative that reduction of PAPR should get the focus rather than taking compensating measures by sacrificing power efficiency.

Because of the reasons mentioned above, PAPR reduction problem has drawn significant research interests and as a result quite a significant number of methods of PAPR reduction can be found in literature. Amongst them signal clipping, selected mapping (SLM) and partial transmit sequence (PTS) are considered by many as the fundamental PAPR reduction methods. In signal clipping method, the peaks in the signal above a certain prescribed level are clipped. This is a very simple technique. But clipping any signal causes in-band distortion and out of band radiation; both resulting in higher BER. On the other hand, SLM and PTS methods are classified as distortion-less phase control schemes. In SLM, multiple signals are generated by multiplying the information bearing signal with pre-decided phase sequences. And then among these multiple generated signals, signal with the lowest PAPR is selected for transmission [4]. Whereas in PTS, the lowest PAPR signal is constructed by optimally phase combining multiple signal sub-blocks [5].

Both SLM and PTS are flexible methods and have good PAPR reduction capability. But they impose added computational overhead on the system. In this respect, the number of different phase sequences (U) in SLM and the number of sub-blocks (V) in PTS are the two parameters that are of importance. While higher values of U and V increase the probability of achieving better PAPR reduction, on the downside they also increase the computational complexity. Again the spectral occupancy for different values of U and V may be different since they essentially represent different signals. The issue of retaining spectral compactness is also very important. Since trading spectral efficiency for PAPR reduction may not be acceptable as it refers to nothing but losing one of the fundamental advantages of OFDM i.e. high spectral efficiency.

In light of all the above mentioned observations, we were motivated to investigate the comparative performance of SLM and PTS from three different perspectives i.e. PAPR reduction ability, computational complexity and finally spectral efficiency. In particular, after determining acceptable values of U and V with reasonable PAPR reduction, we compare SLM and PTS on their spectral performance and recommend PTS over SLM.

II. PEAK TO AVERAGE POWER RATIO (PAPR)

The Peak to Average Power Ratio (PAPR) of OFDM is defined as the ratio between the maximum instantaneous power and the average power. The PAPR (in dB) of the transmitted OFDM signal can be written in mathematical expression as:

\[
PAPR \ (dB) = 10 \log_{10} \max \left( \frac{\|X[n]\|^2}{E[\|X[n]\|^2]} \right), \ 0 \leq t \leq T
\]  
(1)
Fig. 1: Block diagram of selective mapping (SLM) technique for PAPR reduction.

Where $X[n]$ denotes an OFDM signal after IFFT and $E[.]$ denotes expectation. The complex time domain OFDM signal for $N$ subcarriers can be represented as

$$X[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x[k] e^{j\frac{2\pi kn}{N}}, 0 \leq n \leq N - 1$$

(2)

However in practice, it is preferred to take the probability of PAPR exceeding a threshold as the measurement index to represent the distribution of PAPR. This is described as Complementary Cumulative Distribution Function (CCDF) and is mathematically expressed as:

$$\Pr(PAPR > z) = 1 - (1 - \exp(-z))^N$$

(3)

Where $z$ is the threshold value and $N$ is the number of subcarrier.

III. PAPR REDUCTION TECHNIQUES

There are many different techniques that have been proposed to provide a solution to the problem of high PAPR of OFDM system. Among these techniques, this paper is focused on Selective Mapping (SLM) and Partial Transmit Sequences (PTS) [6]. These are discussed below.

A. SELECTED MAPPING (SLM)

The main objective of this technique is to generate a set of data blocks at the transmitter end which represent the original information and then to choose the most favorable block among them for transmission. Fig. 1 shows the block diagram of selective mapping (SLM) technique for PAPR reduction.

Here, the input data block $X = [X[0], X[1], \ldots, X[N-1]]$ is multiplied with $U$ different phase sequences $P^u = [e^{j\phi_{u0}}, e^{j\phi_{u1}}, \ldots, e^{j\phi_{u(N-1)}}]^T$ where $\phi_{u0} = 0$ and $\phi_{u0} \in [0,2\pi)$ for $v=0,1,\ldots,N-1$ and $u=1,2,\ldots,U$, which produce a modified data block $X^u = [X^u[1], X^u[2], \ldots, X^u[N-1]]^T$. IFFT of $U$ Independent sequences $X^u[v]$ are taken to produce the sequences $x^u = [x^u[0], x^u[1], \ldots, x^u[N-1]]^T$ among which the one $\tilde{x} = x^{u_{\text{max}}}$ with the lowest PAPR is selected for transmission, as shown as

$$\tilde{u} = u = 1,2,\ldots,U \left( \max_{u=1,2,\ldots,U} \left( \sum_{n=0}^{N-1} \left| x^{u[n]} \right|^2 \right) \right)$$

(4)

In order for the receiver to be able to recover the original data block, the information (index $u$) about the selected phase sequence $P^u$ should be transmitted as side information. PAPR is decrease when the number of sequence increases.

B. PARTIAL TRANSMIT SEQUENCE (PTS)

In the partial transmit sequence (PTS) technique an input data block of $N$ symbols is partitioned into $V$ disjoint sub-blocks as follows:

$$X = [X^0, X^1, X^2, \ldots, X^{V-1}]^T$$

(5)

Where $X^i$ are the sub-blocks that are consecutively located and also are of equal size. Unlike the SLM technique in which scrambling is applied to all subcarriers, scrambling (rotating its phase independently) is applied to each sub-block in the PTS technique (see Fig. 2). Then each partitioned sub-block is multiplied by a corresponding complex phase factor $b^v, v = 1,2,\ldots,V$, subsequently taking its IFFT to yield

$$x = IFFT \left( \sum_{v=1}^{V} b^v X^v \right) = \sum_{v=1}^{V} b^v \cdot IFFT(X^v)$$

(6)

Where $X^v$ is referred to as a partial transmit sequence (PTS). The phase vector is chosen so that the PAPR can be minimized, which is shown as

$$[\tilde{b}^1, \ldots, \tilde{b}^V] = \arg\max_{[b^1, \ldots, b^V]} \left( \sum_{n=0}^{N-1} \left| \sum_{v=1}^{V} b^v x^{v[n]} \right|^2 \right)$$

(7)

Then, the corresponding time-domain signal with the Table 1: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of OFDM symbol</td>
<td>10000</td>
</tr>
<tr>
<td>Number of Subcarrier</td>
<td>64</td>
</tr>
<tr>
<td>Number of data per subcarrier</td>
<td>52</td>
</tr>
<tr>
<td>Modulation</td>
<td>4QAM</td>
</tr>
<tr>
<td>Channel</td>
<td>AWGN</td>
</tr>
<tr>
<td>Over-sampling factor(L)</td>
<td>4</td>
</tr>
<tr>
<td>Number of Sub-block(V)</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>Number of sequence(U)</td>
<td>2, 4, 8, 16</td>
</tr>
</tbody>
</table>
Fig. 4: Spectral Occupancy or Out of Band radiation of SLM technique with N=64

The lowest PAPR vector can be expressed as:

$$\vec{s} = \sum_{p=1}^{V} b^p \vec{x}^p$$

(8)

The PTS technique requires V IFFT operations for each data block and $\log_2 W^V$ bits of side information [7].

IV. COMPUTATIONAL COMPLEXITY

Computational complexity depends on number of IFFT operations in OFDM signal. A total $V W^V$ number of IFFT operations performed in PTS technique, where V is the number of sub-block and W is the possible variations of the phase. In SLM, every time when SLM technique is applied, it requires calculating the U group IFFTs at the transmitter compared to only one on ordinary OFDM system and its U of N points IFFTs operation needs $n_{mul} = U \frac{N}{2} \log_2 N$ complex multiplication and $n_{add} = U \frac{N}{2} \log_2 N$ addition separately. These problems usually pose high difficulties on real OFDM implementation; therefore, it is required to reduce the computational complexity [6].

V. SIMULATION RESULTS AND DISCUSSION

For comparative performance analysis, we performed simulation based on the model shown in Figs. 1 and 2. We considered QAM modulation and investigated scenarios for sequence U= 2, 4, 8, 16 and sub-block V=2, 3, 4. All the pertinent simulation parameters are listed in Table I.

Figure 3 shows the performance of SLM PAPR reduction technique for different values of sequence U with subcarrier size N=64. From this figure it is observed that SLM technique reduces PAPR significantly compared to the original OFDM signal. When the number of sequence U is increased, the PAPR decreases. But with increasing U, the computational complexity also increases. However, it is difficult to achieve linear growth of PAPR reduction performance with further improvement of U (like U>=8).

$$\text{Table 2: Computational Complexity of SLM technique}$$

<table>
<thead>
<tr>
<th>Sequence, U</th>
<th>Computational Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>768</td>
</tr>
<tr>
<td>4</td>
<td>1536</td>
</tr>
<tr>
<td>8</td>
<td>3072</td>
</tr>
<tr>
<td>16</td>
<td>6144</td>
</tr>
</tbody>
</table>

Figure 4 shows the spectral occupancy or out of band radiation performance of SLM technique for different value of sequence U with subcarrier size N=64. Here, it is observed that, the spectral occupancy performance of SLM method improves for the value of U (like U>=8).

From the above discussion, we understand that if computational complexity is not taken into consideration, higher values of U e.g. U=16 serve the purpose. But U=16 has high computational complexity. In addition, with U=16 we get PAPR reduction is not significantly high when compared to the high computational complexity. Hence we preferred lower value of U i.e. U<=8, so as to avoid introducing too much computational complexity.

Figure 5 shows the performance of PTS technique for different number of sub-blocks V with subcarrier size N=64. From the Fig. 5, it can be observed that increasing sub-block number V leads to the improvement of PAPR reduction performance. Sub-block, V=4 shows better PAPR performance than V=2 and 3.

Figure 6 shows the Spectral Occupancy performance of PTS technique for different value of sub-block with subcarrier size N=64. From Fig. 6, it can be observed that PTS has significant out of band radiation improvement for sub block V=4 with compare to original OFDM signal.

From the above observation we can decide that in practical applications, it is preferred to take sub block number V=4 which gives better PAPR and spectral occupancy performance.

Figure 7 shows the performance of PTS and SLM PAPR reduction technique for same values of sequence U and sub-block V as 4 with subcarrier size N=64. From Fig. 7 it can be seen that PTS has significant out of band radiation improvement for sub block V=4 which gives better PAPR and spectral occupancy performance.

Figure 8 shows the performance of PTS and SLM PAPR reduction technique for same values of sequence U and sub-block V as 4 with subcarrier size N=64. From Fig. 8 it can be seen that PTS has significant out of band radiation improvement for sub block V=4 which gives better PAPR and spectral occupancy performance.
observed that PTS method has significant PAPR reduction with compare to original OFDM signal. When both U and V are same, PTS technique give better performance than SLM technique. For the both same number of U and V=4 PTS techniques gives 0.84dB PAPR reduction with compare to SLM.

Figure 8 shows the spectral occupancy or out of band radiation performance of PTS and SLM PAPR reduction technique for same value of sequence U and sub-block V as 4 with number of subcarrier N=64. From Fig. 8, it can be observed that PTS method has significant out-of-band radiation reduction compared to the original OFDM signal. SLM method has more out of band radiation than original OFDM signal. An improved 3.8 dB out of band radiation improved in PTS with compared to original OFDM signal.

From the above discussion of PTS and SLM PAPR reduction technique, it can be decide that with same value of U and V PTS method has better performance. It is preferred to use PTS PAPR reduction technique for practical application.

VI. CONCLUSION

In this paper, we have investigated the result obtained from applying PAPR reduction technique for different value of sequence number U in SLM and sub-block number V in PTS. The performance parameters are PAPR, computational complexity and spectral occupancy. In SLM PAPR reduction technique of OFDM system, it has been observed that if computational complexity is considered, SLM technique shows better performance for sequence U<=8. If computational complexity is not considered, SLM technique with sequence U=16 shows better performance. In PTS PAPR reduction technique of OFDM system, it has been observed that, PTS PAPR reduction technique shows better performance for sub-block number V=4. Computational complexity is taken into consideration. With knowing that PTS method shows better performance for V=4 and SLM method for U<=8, we compare

PTS and SLM method. PTS method shows better PAPR, computational complexity and spectral occupancy or out of band radiation performance than SLM. PTS method can be considered because of better spectral compactness or lower computational complexity and less out of band radiation. Therefore, PTS method is the best between PTS and SLM. However, the transmitter and receiver complexity is very high. Thus in practical applications, a tradeoff needs to be made between good performance and auxiliary information. From the above made discussion, SLM technique is more suitable if system can tolerate more redundant information; otherwise, PTS technique is more acceptable when complexity becomes the first considering factor.

REFERENCES