Progressive failure analysis of slopes using non-vertical slices

Younus Ahmed KHAN*, Jing-Cai JIANG** and Takuo YAMAGAMI**

Abstract
A limit equilibrium method for progressive failure analysis of slopes is proposed based on a technique of non-vertical slices. The local factors of safety are calculated by defining variable factor of safety instead of conventional single value factor of safety along a shear surface. Reasonable simplifying assumptions about the inter-slice forces and the line of thrust are made to render the problem statically determinate. Strain softening, though in an approximate manner, is considered in the solution procedure. The analysis of a field case history indicates that the proposed method realistically represent the behaviour of progressive failure of an actual slope. Another case study revealed that in certain conditions a reasonable solution can be obtained using non-vertical slice approach, but the vertical slice analysis does not work well in obtaining reliable results. The proposed method, which can also handle vertical slice division, enables the user to choose more appropriate results either from vertical or from non-vertical slice analyses.

Key words : slope stability, progressive failure, local factor of safety, limit equilibrium, non-vertical slice

1. Introduction
An actual failed slope generally exhibits the behaviour of progressive failure: local yielding or failure initiated at some points gradually develops and finally leads to overall failure of the slope along a slip surface. Therefore, a realistic analysis of slope stability should include the effect of such a process of progressive failure. The phenomenon of progressive failure of slopes was investigated early in 1960s (e.g. Skempton, 1964; Bishop, 1967; Bjerrum, 1967). Numerical analysis techniques, such as finite element method (FEM) (e.g. Lo & Lee, 1973; Potts et al., 1990), have been a useful tool for analysis of progressive failure in slopes. However, there are often difficulties in applying such complicated techniques: the authors thus believe that it is still necessary to develop a simple method of stability analysis to account for the nature of progressive failure of slopes.

Limit equilibrium procedures have been widely used in conventional slope stability analyses. A number of attempts such as Law and Lumb, 1978; Chugh, 1986; Srbulov, 1987 have been made to consider the process of progressive failure based on the limit equilibrium concept. In the method of Law and Lumb, all the interslice forces are ignored. The solution procedure in the Srbulov's method is somewhat redundant as the numbers of unknowns and equations are not equal. Chugh introduced one unknown scalar factor in his method, which seems to be difficult to determine. The above-mentioned methods are thus, not so satisfactory.

A limit equilibrium approach was developed (Yamagami & Taki, 1997; Yamagami et al., 1999) to carry out progressive failure analysis of slopes. The problem was made determinate well by simultaneously using the simplifying assumptions of the inter-slice forces and the line of thrust which were separately employed in Morgenstern-Price method (1965) and the Janbu method (1954). The behaviour of soil softening was also taken into account in an approximate manner. The effectiveness of the method has been verified through a number of case studies.

All the aforementioned methods were presented using vertical slices. In principle, however, stability analysis should be performed using both vertical and non-vertical slice divisions on each slip surface to find the smallest factor of safety. Furthermore, it is necessary to check the physical acceptability of the solution, that is, the internal forces obtained from the solution must not violate failure criteria and tension must not be implied within the soil mass. Studies suggested that the vertical slice interfaces are sometimes not suitable for an evaluation of internal forces (Sarma, 1979). For complex problems the conventional vertical slice approach has often experienced difficulties with solution instabilities (Donald & Chen, 1999).

This paper proposes a limit equilibrium analysis of progressive failure of slopes based on non-vertical slices to meet the general demand mentioned above.
and to overcome the difficulties involved in using vertical slices. As done in Yamagami et al. (1999), a local factor of safety is defined at the base of each non-vertical slice, and then, reasonable simplifying assumptions about the interslice forces and the line of thrust are made to determine the local factors of safety along the slip surface. Strain softening is also considered in an approximate manner. Application of the proposed method to a highly non-circular slip surface showed that the reliable results were obtained from non-vertical slice approach, but the analysis using vertical slices completely failed to determine a reasonable solution. It should be pointed out that the proposed method can also handle vertical slices. Therefore, the method is more general and provides the alternative to select the best results either from vertical or from non-vertical slice analyses.

2. Method of analysis

2.1 Necessary assumptions

In Fig.1(a), the body of mass contained within the assumed slip surface and the ground surface is divided into \( n \) non-vertical slices. The forces acting on an inclined slice are given in Fig.1(b). \( E_i \) and \( X_i \) are the normal and shear forces acting on the slice interface respectively; \( Z \) is the length between acting point of \( E_i \) and the point \( C \) at the lower left corner of slice; \( \delta_i \) is the inclination of slice interface with y-axis; \( \alpha_i \) is the angle between slice base and x-axis; \( N_i \) and \( T_i \) are the total normal force and shear force acting on the slice of the slice respectively; \( l_i \) is the length of the base slice; \( b_i \) is the horizontal length of slice base; and \( d_i \) is the length between the point \( C \) and the acting point of \( N_i \) at the slice base. The weight of the slice (\( W_i \)) and the horizontal earthquake acceleration (\( K_i \)) are acting at the center of gravity \((x_{gi}, y_{gi})\) of the slice.

For such a situation, we have the following unknowns: \( n \) number of \( N \) force, \( n \) number of location of \( N \) force, \( n \) number of \( E \) force, \( n - 1 \) number of location of \( E \) force and \( n - 1 \) number of \( X \) force. Because we define a local factor of safety at each slice base in the present method, another \( n \) number of unknowns for local factor of safety is added to the unknown list. The total unknowns become \( 7n - 3 \). On the other hand, there are following \( 4n \) number of equations: \( n \) number of horizontal force equilibrium equations; \( n \) number of vertical force equilibrium equations; \( n \) number of moment equilibrium equations and \( n \) number of factor of safety equations at the base of each slice. Therefore, the numbers of required assumptions are \( 3n - 3 \) in order to make the problem determinate. We assume \( n \) acting points of \( N \) forces, \( n - 1 \) acting points of \( E \) forces (Janbu, 1954) and \( n - 1 \) relationships between \( X \) and \( E \) forces, i.e. \( X = a \cdot f(x) \cdot E \) (Morgenstern-Price, 1965). The assumption of \( X = a \cdot f(x) \cdot E \) leads to introduction of one extra unknown \( a \). Therefore, the number of unknowns is reduced to \( 7n - 2 \), which is the same as the number of equations. The problem is now determinate (see Table 1), and local factors of safety can be calculated.

2.2 Formulation of equations

If the Mohr-Coulomb model is valid for describing the shear strength of soil, the local factor of safety equation at the base of each slice can be expressed by

\[
T_i = \frac{1}{F_i} \left[ c_i l_i + (N_i - u_i l_i) \tan \phi_i \right] \tag{1}
\]

where, \( c_i, \phi_i \) are the strength parameters, \( u_i \) is the pore pressure and \( F_i \) is the local factor of safety of the slice.

Resolving the forces vertically and horizontally we have.

---

**Table 1** Number of unknowns and equations.

<table>
<thead>
<tr>
<th>Unknowns</th>
<th>Equations/Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local factor of safety</td>
<td>( n )</td>
</tr>
<tr>
<td>Normal force, ( N )</td>
<td>( n )</td>
</tr>
<tr>
<td>Location of ( N ) force</td>
<td>( n )</td>
</tr>
<tr>
<td>Shear force, ( T )</td>
<td>( n )</td>
</tr>
<tr>
<td>Normal force, ( E )</td>
<td>( n-1 )</td>
</tr>
<tr>
<td>Location of ( E ) force</td>
<td>( n-1 )</td>
</tr>
<tr>
<td>Shear force, ( X )</td>
<td>( n-1 )</td>
</tr>
<tr>
<td>A parameter, ( A )</td>
<td>1</td>
</tr>
<tr>
<td>Total unknowns</td>
<td>( 7n-2 )</td>
</tr>
<tr>
<td>Total equations</td>
<td>( 7n-2 )</td>
</tr>
</tbody>
</table>
From equations (1) and (2), we get,

\[ T_i \sin \alpha_i + N_i \cos \alpha_i = W_i - E_{i+1} \sin \delta_{i+1} + X_{i+1} \cos \delta_{i+1} + E_{i} \sin \delta_i - X_i \cos \delta_i \] 

(2)

\[ T_i \cos \alpha_i - N_i \sin \alpha_i = K_i W_i + E_{i+1} \cos \delta_{i+1} + X_{i+1} \sin \delta_{i+1} - E_{i} \cos \delta_i - X_i \sin \delta_i \] 

(3)

By combining the equations (2), (3) and (4) for eliminating \( T_i \) and \( N_i \), we have,

\[ \begin{aligned}
A_1 &= m_2 \sin \alpha_i + m_1 \cos \alpha_i \\
A_2 &= m_2 \sin \alpha_i + m_1 \cos \alpha_i \\
A_3 &= m_2 \cos \alpha_i - m_1 \sin \alpha_i \\
A_4 &= m_2 \cos \alpha_i + m_1 \sin \alpha_i \\
A_5 &= (m_2 \sin \alpha_i - m_1 \cos \alpha_i) \\
A_6 &= K_i W_i m_1 \\
A_7 &= m_2 \tan \phi_i + m_1 \tan \phi_i \\
A_8 &= m_2 \tan \phi_i - m_1 \tan \phi_i \\
\end{aligned} \]

(4)

where,

\[ A_1 = m_2 \sin \alpha_i + m_1 \cos \alpha_i \]
\[ A_2 = m_2 \sin \alpha_i + m_1 \cos \alpha_i \]
\[ A_3 = m_2 \cos \alpha_i - m_1 \sin \alpha_i \]
\[ A_4 = m_2 \cos \alpha_i + m_1 \sin \alpha_i \]
\[ A_5 = (m_2 \sin \alpha_i - m_1 \cos \alpha_i) \]
\[ A_6 = K_i W_i m_1 \]
\[ A_7 = m_2 \tan \phi_i + m_1 \tan \phi_i \]
\[ A_8 = m_2 \tan \phi_i - m_1 \tan \phi_i \]

To solve the equation (5), we assume, as mentioned before, the relation between inter-slice force \( E \) and \( X \), which is similar to that of Morgenstern-Price method.

\[ X = \lambda f(x)E \] 

(6)

where \( \lambda \) is an unknown parameter to be determined, and \( f(x) \) is a known function.

Therefore, putting the value of \( X_i \) and \( X_{i+1} \) from equation (6) we obtain a recurrence equation (7) of inter-slice forces.

\[ E_{i+1} = E_{i} \left[ A_2 - \frac{\lambda f(x)A_4}{A_1 - \lambda f(x)A_3} \right] + \frac{A_5 - A_6}{A_1 - \lambda f(x)A_3} \] 

(7)

Considering moment equilibrium (\( M_i = 0 \)) about the left corner-point, \( C(xb_i, yb_i) \) of the base of the slice, we obtain,

\[ N_i d_i + E_{i+1} [Z_{i+1} + b \sec \alpha_i \sin \alpha_i + \delta_{i+1}] - E_{i} Z_i - X_i \sec \alpha_i \cos \alpha_i + \delta_{i+1} + K_i W_i (y_i - yb_i) - W_i (x_i - xb_i) = 0 \] 

(8)

Now putting \( X_{i+1} = \lambda f(x)E_{i+1} \) into equation (8), we have the following equation of moment equilibrium:

\[ N_i d_i + E_{i+1} [Z_{i+1} + b \sec \alpha_i \sin \alpha_i + \delta_{i+1}] - \lambda f(x)E_{i+1} \sec \alpha_i \cos \alpha_i + \delta_{i+1} - E_{i} Z_i - X_i \sec \alpha_i \cos \alpha_i + \delta_{i+1} + K_i W_i (y_i - yb_i) - W_i (x_i - xb_i) = 0 \] 

(9)

In equation (9) \( \lambda \) and \( F_i \) are the only unknowns, since a value for \( E_{i+1} \) is determined from equation (7) with a previously determined value of \( E_i \) and then \( N_i \) is calculated using the equation (4). Hence the equation (9) is re-written as,

\[ M_i [\lambda, F_i] = 0 \] 

By solving the equation (10) (for details see the section 2.3) with an iterative method, for example the Secant method (e.g. Jain, et al. 1993), the moment equilibrium for individual slice is satisfied and the value of \( F_i \) \( (i = 1, \cdots, n) \) in sequence can be obtained. The complete solution must satisfy the boundary condition, \( E_{n+1} = 0 \) at the end of the slip surface.

2.3 Calculation procedures

The calculation procedures for local factors of safety of a slope, which is divided into \( n \) non-vertical slices numbering 1 to \( n \) from left to right, are as follows:

1. Set one initial value of \( \lambda (= \lambda_1) \) and the assumed known values of \( f(x_i), Z_i \), and \( d_i \) for \( n \) slices.

2. Assume two initial \( F \) values, \( F_{i1} \) and \( F_{i2} \) of slice \( i \) for Secant method.

3. Starting with \( F_{i1} \), calculate \( X_{i+1} \) and \( E_{i+1} \) from equations (6) and (7) respectively.

4. Find \( N_i \) value from equation (4) and moment value \( M_{i1}[\lambda_1, F_{i1}] \) from equation (9).

5. Putting another initial \( F (= F_{i2}) \) value and recalculate \( M_{i2}[\lambda_1, F_{i2}] \) as,

\[ M_{i2}[\lambda_1, F_{i2}] = \frac{F_{i1} M_{i1}[\lambda_1, F_{i1}] - F_{i2} M_{i1}[\lambda_1, F_{i1}]}{F_{i1} - F_{i2}} \] 

(11)

If \( (F_{i1} - F_{i2}) > \text{tolerance} \), then recalculate \( M_{i1}[\lambda_1, F_{i1}] \) with this \( F_{i2} \) and find next \( F_{i:n+1} \) until \( (F_{i:n+1} - F_{i:n}) < \text{tolerance} \) is satisfied. So, \( F_i \) is taken to be \( F_{i:n+1} \) if \( (F_{i:n+1} - F_{i:n}) < \text{tolerance} \). Here, \( F_{i:n+1} \) is the immediately previous value of \( F_{i:n} \) and tolerance is \( 1.0 \times 10^{-6} \).

7. Repeating the processes from 2 to 6 of Step-II, for \( n \) number of slices we find the values of \( F_1, F_2, \cdots, F_n \). Now, check the boundary condition, \( E_{n+1} = 0 \) using equation (7); usually \( E_{n+1} \neq 0 \).
8. Go to the step-I and set the another initial value of $\lambda (=\lambda_2)$ and again check the boundary condition, $E_{n+1}=0$; if it is not satisfied change the $\lambda$ value to $\lambda_{new}$. $\lambda_{new}$ is given as (secant method),

$$
\lambda_{new} = \frac{\lambda_1 E_{n+1} \{F_2, ..., F_6\} - \lambda_2 E_{n+1} \{\lambda_1, F_1, ..., F_4\}}{E_{n+1} \{F_2, ..., F_6\} - E_{n+1} \{\lambda_1, F_1, ..., F_4\}} \ldots (12)
$$

if $(\lambda_{new} - \lambda_{new-1}) < \text{tolerance}$, then $\lambda_{new} = \text{final}\ \lambda$ value, if not reiterate the processes from 1 to 8 of Step-II with the successive $\lambda_{new}$ until it is satisfied. $\lambda_{new-1}$ is the immediately previous value of $\lambda_{new}$ and tolerance $= 1.0 \times 10^{-6}$

9. At this stage, we get a set of $F$ values after satisfying all the conditions. This set of $F$ values represents the local factors of safety of the slices.

2.4 Optimization of $f(x)$, $Z$ and $d$

In the Morgenstern-Price method, $f(x)$ is taken as a simple known function, for example, a constant (e.g. 1) or half sine and so on. In the Janbu method, $Z$ is assumed usually to be 1/3 of the inter-slice boundary. And $d$ is generally assumed to be at the center of slice base. Using these assumptions, the present procedure may not be converged. In vertical slice cases, it has been indicated that $f(x)$ and $Z$ must be optimized to obtain a complete converged solution (Yamagami, et. al. 1999). A similar optimization procedure is thus constructed for the present study. In addition to $f(x)$ and $Z$, $d$ is also included in the optimization because the slice width may not always be small in the non-vertical slices analysis. So, the boundary condition can be reached by optimizing the following equation:

$$
|E_{n+1}|^2 = F_{\text{un}} \{f(x_0), ..., f(x_{n-1}), Z_{0}, ..., Z_{n-1}, d_0, ..., d_n\} \rightarrow \text{minimize} (= 0) \ldots \ldots \ldots \ldots (13)
$$

The Nelder-Mead simplex method (Nelder & Mead, 1965) for non-linear programming is applied to solve the equation (13).

2.5 Considering softening

In a slope, after the development of softening in certain part of the potential failure surface, the progressive failure would initiate, so that this fact must be considered in the calculation process. As the softening cannot be defined with the amount of deformation or strain in the limit equilibrium analysis, it is assumed that the soil resistance will drop abruptly to the final residual value (as Law and Lamb, 1978) immediately after reaching the peak value (Fig.2).

Peak strength ($R_0$) and Residual-strength ($R_r$) are expressed as,

$$
R_0 = c_l + N \tan \phi \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (14)
$$

$$
R_r = c_1 + N \tan \phi_r \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (15)
$$

where $c_l$ and $\phi_l$ are peak strength parameters and $c_r$, $\phi_r$ are residual ones.

The softening processes are included by the following iterative procedures:

(a) At the beginning, every slice is assigned with the peak strength.

(b) The local factors of safety are calculated using the calculation procedures discussed in the previous section.

(c) If slices whose local factor of safety is less than one ($F < 1$) emerge, the peak strength of such slices is then replaced by the residual strength.

(d) The calculation procedure is continued until the peak strength of all the slices with $F < 1$ are replaced with residual strength. This means that the factors of safety of the slices still having the peak strength are all greater than unity. In the case of no softening, the first two steps (a) and (b) result in the convergent solution directly.

2.6 Overall factor of safety

For evaluating the stability of the slope as a whole, we define an overall factor of safety, $F_{\text{overall}}$ by the ratio between the sum of the mobilized shear forces ($T$) and the sum of the available shear strengths ($R_0$ and $R_r$) along the entire slip surface.

$$
F_{\text{overall}} = \frac{\sum_{i=1}^m R_0 + \sum_{i=1}^m R_r}{\Sigma T} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (16)
$$

where $m$ is the number of slices with residual strength among the total slices ($n$).

3. Techniques of AILC & AGLC

In the above-mentioned solution procedure, the local factors of safety (L.Fs.S.) of the slices in local failure zones are allowed to be less than unity even if the overall factor of safety of the slope itself is much greater than unity. This solution procedure is termed as AILC (Analysis of Instantaneous Loading Condition) in Yamagami et al.’s paper (1999). If L.Fs.S. of a part of
Khan, Y.A., Jiang, J-C. and Yamagami, T.: Progressive failure analysis of slopes using non-vertical slices

the slip surface, obtained from the AILC, are below or at unity, then it indicates that the local failure has occurred in that part of the slip surface. Even in such cases, sliding (overall failure) will not necessarily take place along the slip surface unless the overall factor of safety is at unity or less than unity. In reality, however, the L.F.s.S. of local failure zones should be at unity (shear stresses being equal to shear strengths) as long as the soil mass above the slip surface is in equilibrium.

To meet this condition, Yamagami et al. (1999) contrived a new solution procedure, AGLC (Analysis of Gradual Loading Condition), in which local factors of safety of locally failed zones are constrained to keep at unity. In the following, a similar AGLC procedure for non-vertical slices is briefly described to deal with situations after local failures occur. Let us have the solution of AILC in which a part of the given slip surface is assumed to be locally failed, as shown in Fig.3(a). That is, the slices to m are assumed to have L.F.s.S. less than or equal to unity as the result of the AILC.

(1) First step

The solution of AGLC is obtained by an iterative procedure starting with the results of AILC. If the value of local factor of safety (L.F.S.) of slice j shown in Fig.3(a) is the smallest of all the L.F.s.S, then the iterative procedure starts with making the factor of safety of slice j equals to unity. More specifically, we treat the L.F.S. for slice j as known (=1.0) whereas the rest is unknown, and similar computations to the AILC procedure described in the preceding section is then performed. The above computations may produce new local failure zones (different from those obtained from the initial AILC procedure) with local factors of safety less than unity.

(2) Second step

Fig.3(b) shows a situation just after the First step where the Fj is equal to unity. Assume that a part p'q'o of the slip surface have locally failed as result of First step. Usually, the part p'q'o does not coincide with the initial failure zone pqo shown in Fig.3(a). Now suppose that slice k has the smallest L.F.S among all the local factors of safety. Then, we conduct a similar computation of AILC with the condition that Fk =1.0 and Fj =1.0 (Fig.3(c)), and a new set of L.F.s.S. will be obtained as a converged solution. Subsequently, a similar iterative computation is done in order to obtain the solution in which, beside the slices j and k, another slice l retains a factor of safety of 1.0. In this way, each local factor of safety becomes equal to unity one by one for the slices in locally failed zones. The computations are continued until no slices occur which have factors of safety smaller than unity. Consequently, the AGLC procedure finally yields a converged solution that all the L.F.s.S. are greater than or equal to unity.

4. Case studies and discussions

We present two example case studies to demonstrate the capability of the proposed method. As this method can also handle the vertical slice pattern, each example was analyzed by dividing the slope into both non-vertical and vertical slices.

4.1 Example 1: Selset landslide, Yorkshire, UK

Selset landslide is a failed valley slope of the River Lune, Yorkshire, which is chosen for the present case study.
study. Skempton & Brown (1961) first analyzed this slope with the Bishop method (Bishop, 1955) but failed to provide the satisfactory results. This is because in the Bishop method the whole slip surface is assumed to have constant strength parameters (i.e. either peak or residual ones). A progressive failure analysis for Selset landslide was carried out using the vertical slices (Yamagami et al., 1999), and the reasonable results were obtained. This example is reanalyzed by dividing the slope into non-vertical slices to show the effectiveness of the proposed method.

Fig.4(a) shows the slope profile of Selset landslide, the division of non-vertical slices and the peak and residual strength parameters used in the analysis. The values of pore pressure ratio, \( \phi \) along the slip surface shown in the figure were obtained from the original flow net profile by Skempton & Brown (1961). First, the slope was analyzed using the AILC procedure where local factors of safety are allowed to be less than unity. From the AILC results shown in Fig.4 (b) it is seen that the L.F.s.s. of the slip surface from the middle to the top of the slope are less than unity. This implies that the slope failure may probably be initiated from somewhere within the local failure zone (the bases of slices No.8 to No.14 in Fig.4(a)). It should be noted that the conventional limit equilibrium procedures with single value factor of safety can not predict this phenomenon of local failure along the shear surface.

As the locally failed zone was found, an AGLC procedure was subsequently carried out. The local and overall factors of safety obtained from the AGLC are shown in Fig.4(b) together with the results from the AILC. Here we observe that the AGLC procedure results in a local factor of safety of unity for almost all the slices and an overall factor of safety of 1.0472. This indicated that the entire slope attained a failure condition as it actually occurred. In other words, the AGLC analysis leads to a realistic solution which reasonably represent the behaviour of progressive failure of Selset landslide.

The proposed method was also used to analyze Selset landslide by dividing the slope into vertical slices. Fig.5(a) shows the division of vertical slices which is the same as that used by Yamagami et al. (1999). The results from the AILC and AGLC procedures are illustrated in Fig.5(b). A comparison between the results shown in Fig.4(b) and Fig.5(b) indicates that the local failure zones obtained from the non-vertical slice analysis agree well with those calculated using vertical slices but there are slight discrepancies between the values of local factors of safety in some areas of the slip surface. It is interested to note that the overall factor of safety (1.0472) of AGLC using non-vertical slices is smaller than that (1.0965) of using vertical slices. It seems that the former can provide a better or at least equal evaluation of the overall factor of safety compared with the vertical slice analysis.
Fig. 5(b) also illustrated the AILC and AGLC results using vertical slice division by Yamagami et al. (1999). It is seen from Fig. 5(b) that the distribution of local failure zones from the proposed method is almost same as that of Yamagami et al.'s analysis. The values of local factors of safety obtained from both methods are in good agreement for the slices in the local failure zones but somewhat different for the other slices.

Effect of the number of slices on calculated results was also examined. When the divided slices exceed a certain number (14 slices in this example), the distribution of local failure zones and values of the local and overall factors of safety are hardly influenced by the number of slices. It has been indicated that this certain number is almost the same as the amount of slices required for conventional limit equilibrium approaches. Due to the limitation of space, no details are given herein.

4.2 Example 2: A Slope with thin weak layer

This is a slope with a thin weak layer, which leads to a highly non-circular failure surface with a major linear portion, as shown in Fig. 6(a). Donald and Giam (1989) analyzed this slope using the GWEDGEM(a Generalized WEDGE Method), and the critical slip surface ABCD shown in Fig. 6(b) was obtained. They also carried out an analysis with FEM to validate their results. An elasto-plastic model using the Mohr-Coulomb failure criterion and the Cambridge CRISP computer program (Gunn and Britto, 1981) were employed for the FE stress-strain calculations. The FEM results are denoted by crosses in Fig. 6(b). A cross in this figure has two implications. Two lines consisting of a cross are inclined at $\pm (45^\circ - \phi_{\text{mob}}/2)$ to the major principal compressive stress, representing potential slip directions at the location of the cross. Note that $\phi_{\text{mob}} = \tan^{-1}(\tan \phi / F)$ is the mobilized angle of friction and $F$ is the factor of safety calculated by the GWEDGEM on the slip surface ABCD. The other implication of the crosses in Fig. 6(b) is that their relative sizes are inversely proportional to the magnitude of the local factors of safety calculated using the FEM stresses. This means that larger size crosses along the slip surface represent lower values of the local factors of safety and vice versa. The FEM results in Fig. 6(b) shows that the values of local factors of safety along the critical slip surface are completely different from one part to another; the smallest (largest size crosses) in the BC section (especially, in the part close to the slope toe) due to low shear strength of the weak layer, smaller in the lower part of the CD section due to relatively high stress levels and largest in the upper part near the ground. As the distribution of local factors of safety along the slip surface is clearly non-uniform, the conventional method with a single value factor of safety may produce unreliable results for such a complex heterogeneous slope.

The proposed method was used to calculate local factors of safety along the slip surface ABCD shown in Fig. 6(b). The division of non-vertical slices and soil parameters used in the calculations are presented in Fig. 6(a). Note that the residual strength parameters of soil #1 and soil #2 (weak layer) are taken to be the same as their peak values in the present study. The results of the proposed method are illustrated in Fig. 6(c). Fig. 7 shows the distribution of interslice forces $E$, $X$ and normal force $N$ on the slip surface. It can be
seen from Fig. 6 that the relative magnitude of local factors of safety obtained from both AILC and AGLC analyses corresponds well to the FE results shown in Fig. 6(b). The local failed zones were found to be along the weak layer, implying that local yielding may be initiated from somewhere within the weak layer. These results indicate that it is very important to perform progressive failure analysis for such a heterogeneous slope since the distribution of local factors of safety in the slope are clearly non-uniform. The overall factor of safety calculated by the proposed method was 1.267, being in good agreement with a factor of safety of 1.27 obtained from the GWEDGEM (Donald and Giam, 1989).

An attempt has been made to analyze the above example by dividing the slope into vertical slices as shown in Fig. 6(a) by vertical broken lines. As a result, the analyses always suffered from negative horizontal stresses, which took place at the vertical slice boundaries. In other words, the statically admissible conditions were not satisfied in the analysis procedure using vertical slices. Therefore, in this case, the analyses using vertical slices completely failed to find a reasonable solution. This indicates that the proposed method using non-vertical slices is, in certain instances, more effective than those based on the analysis using vertical slices.

5. Conclusions

A method of progressive failure analysis of slopes using non-vertical slices has been proposed. A local factor of safety is assigned at the base of each slice within the limit equilibrium concept. By introducing the simplifying assumptions about the inter-slice forces and the line of thrust used separately in the Morgenstern-Price and Janbu methods, the problem has been completely rendered statically determinate. Softening effects of the slope materials can be included by means of residual strength parameters during the solution procedures. An optimized computation procedure was presented to obtain reliable, completely converged solutions.

The analysis of a field case history indicated that a realistic simulation of the actual slope failure was obtained from the proposed method. In the case of the slope with a weak layer, a reasonable solution was attained using non-vertical slice approach. However, the analysis based on vertical slice pattern did not work well to produce the reliable results due to appearance of negative horizontal inter-slice forces. This signifies that the proposed method using non-vertical slices, in certain instances is more effective than that based on vertical slice analysis. Therefore, the method can provide an alternative way to select the best results either from vertical or from non-vertical slice analyses.

References

Law, K. T. and Lumb, P. (1978): A limit equilibrium analysis of
(Received August 3, 2001, Accepted May 17, 2002)