The input circuit appears as a resistance $h_{11}$ in series with a voltage generator $h_{12} v_2$. The output circuit involves two components; a current generator $h_{21} i_1$ and shunt resistance $h_{22}$.

Performance of a Linear Circuit in h Parameters

(i) Input Impedance:

$$Z_{in} = \frac{v_1}{i_1}$$

Now $v_1 = h_{11} i_1 + h_{12} v_2$ in terms of $h$ parameters. Substituting the value of $v_1$ in the above expression, we get,

$$Z_{in} = \frac{h_{11} i_1 + h_{12} v_2}{i_1} = h_{11} + \frac{h_{12} v_2}{i_1}$$

Now, $i_2 = h_{21} i_1 + h_{22} v_2$ in terms of $h$ parameters, and from the figure $i_2 = -v_2/r_L$. So,

$$\therefore \quad \frac{-v_2}{r_L} = h_{21} i_1 + h_{22} v_2 \quad \therefore i_2 = \frac{-v_2}{r_L}$$

or

$$-h_{21} i_1 = h_{22} v_2 + \frac{v_2}{r_L} = v_2 \left( h_{22} + \frac{1}{r_L} \right)$$

$$\therefore \quad \frac{v_2}{i_1} = \frac{-h_{21}}{h_{22} + \frac{1}{r_L}}$$

Substituting the value of $v_2/i_1$ from exp. (ii) into exp. (i), we get,

$$Z_{in} = h_{11} - \frac{h_{12} h_{21}}{h_{22} + \frac{1}{r_L}}$$

...(iii)

If either $h_{12}$ or $r_L$ is very small, the second term in exp. (iii) can be neglected and input impedance becomes:

$$Z_{in} \approx h_{11}$$
(ii) Current gain: referring to the figure, the current gain $A_i$ of the circuit is given by:

$$A_i = \frac{i_2}{i_1}\,$$

Now

$$i_2 = h_{21}i_1 + h_{22}v_2$$

and

$$v_2 = -i_1r_L$$

∴

$$i_2 = h_{21}i_1 - h_{22}i_1r_L$$

or

$$i_2(1 + h_{22}r_L) = h_{21}i_1$$

or

$$\frac{i_2}{i_1} = \frac{h_{21}}{1 + h_{22}r_L}$$

But $i_2/i_1 = A_i$, the current gain of the circuit.

∴

$$A_i = \frac{h_{21}}{1 + h_{22}r_L}$$

If $h_{22}r_L << 1$, then $A_i \approx h_{21}$.

The expression $A_i \approx h_{21}$ is often useful. To say that $h_{22}r_L << 1$ is the same as saying that $r_L << 1/h_{22}$. This occurs when $r_L$ is much smaller than the output resistance ($1/h_{22}$), shunting $h_{21}i_1$ generator. Under such condition, most of the generator current bypasses the circuit output resistance in favour of $r_L$. This means that $i_2 \approx h_{21}i_1$ or $i_2/i_1 \approx h_{21}$.

(iii) Voltage gain:

Referring to the figure again, the voltage gain of the circuit is given by:

$$A_v = \frac{v_2}{v_1}$$

$$= \frac{v_2}{i_1Z_{in}} \quad (\because v_1 = i_1Z_{in}) \quad \ldots (iv)$$

While developing expression for input impedance, we found that:

$$\frac{v_2}{i_1} = -\frac{h_{21}}{h_{22} + \frac{1}{r_L}}$$

Substituting the value of $v_2/i_1$ in exp. $(iv)$, we get,

$$A_v = -\frac{h_{21}}{Z_{in}\left(h_{22} + \frac{1}{r_L}\right)}$$

(iv) **Output Impedance.** In order to find the output impedance, remove the load $r_L$, set the signal voltage $v_1$ to zero and connect a generator of voltage $v_2$ at the output terminals. Then $h$ parameter equivalent circuit becomes as shown in Fig. 24.8. By definition, the output impedance $Z_{out}$ is

$$Z_{out} = \frac{v_2}{i_2}$$

![Fig. 24.8](image-url)
With $v_1 = 0$ and applying Kirchhoff's voltage law to the input circuit, we have,
\[ 0 = i_1 h_{11} + h_{12} v_2 \]
\[ \therefore \quad i_1 = -\frac{h_{12}}{h_{11}} v_2 \]

Now
\[ i_2 = h_{21} i_1 + h_{22} v_2 \]

Putting the value of $i_1 = -\frac{h_{12}}{h_{11}} v_2$ in this equation, we get,
\[ i_2 = h_{21} \left( -\frac{h_{12} v_2}{h_{11}} \right) + h_{22} v_2 \]

or
\[ i_2 = -\frac{h_{21} h_{12} v_2}{h_{11}} + h_{22} v_2 \]

Dividing throughout by $v_2$, we have,
\[ \frac{i_2}{v_2} = -\frac{h_{21} h_{12}}{h_{11}} + h_{22} \]

\[ \therefore \quad Z_{out} = \frac{v_2}{i_2} = \frac{1}{h_{22} - \frac{h_{21} h_{12}}{h_{11}}} \]

**Transistor h parameters**

For small signal transistor behaves like a linear circuit.

\[ v_1 = h_{11} i_1 + h_{12} v_2 \]
\[ i_2 = h_{21} i_1 + h_{22} v_2 \]

The following points are worth noting while considering the behaviour of transistor in terms of $h$ parameters:

(i) For small a.c. signals, a transistor behaves as a linear circuit. Therefore, its a.c. operation can be described in terms of $h$ parameters.

(ii) The value of $h$ parameters of a transistor will depend upon the transistor connection (i.e. CB, CE or CC) used. For instance, a transistor used in CB arrangement may have $h_{11} = 20 \, \Omega$. If we use the same transistor in CE arrangement, $h_{11}$ will have a different value. Same is the case with other $h$ parameters.

(iii) The expressions for input impedance, voltage gain etc. derived in Art. 24.4 are also applicable to transistor amplifier except that $r_L$ is the a.c. load seen by the transistor i.e.
\[ r_L = R_c \ll R_L \]

(iv) The values of $h$ parameters depend upon the operating point. If the operating point is changed, parameter values are also changed.

(v) The notations $v_1, i_1, v_2$ and $i_2$ are used for general circuit analysis. In a transistor amplifier, we use the notation depending upon the configuration in which transistor is used. Thus for CE arrangement,
\[ v_1 = V_{be} \quad ; \quad i_1 = I_b \quad ; \quad v_2 = V_{ce} \quad ; \quad i_2 = I_c \]

Here $V_{be}$, $I_b$, $V_{ce}$ and $I_c$ are the R.M.S. values.
Table below shows the \( h \) parameter nomenclature of a transistor:

<table>
<thead>
<tr>
<th>S.No.</th>
<th>( h ) parameter</th>
<th>Notation in CB</th>
<th>Notation in CE</th>
<th>Notation in CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( h_{11} )</td>
<td>( h_{1b} )</td>
<td>( h_{1e} )</td>
<td>( h_{1e} )</td>
</tr>
<tr>
<td>2.</td>
<td>( h_{12} )</td>
<td>( h_{rb} )</td>
<td>( h_{re} )</td>
<td>( h_{re} )</td>
</tr>
<tr>
<td>3.</td>
<td>( h_{21} )</td>
<td>( h_{jb} )</td>
<td>( h_{je} )</td>
<td>( h_{je} )</td>
</tr>
<tr>
<td>4.</td>
<td>( h_{22} )</td>
<td>( h_{ob} )</td>
<td>( h_{oe} )</td>
<td>( h_{oe} )</td>
</tr>
</tbody>
</table>

Transistor Circuit Performance in \( h \) Parameters

\( i \) Input impedance. The general expression for input impedance is

\[
Z_{in} = h_{11} - \frac{h_{12} \cdot h_{21}}{h_{22} + \frac{1}{r_L}}
\]

Using standard \( h \) parameter nomenclature for transistor, its value for CE arrangement will be:

\[
Z_{in} = h_{1e} - \frac{h_{re} \cdot h_{je}}{h_{oe} + \frac{1}{r_L}}
\]

\( ii \) Current gain. The general expression for current gain is

\[
A_I = \frac{h_{21}}{1 + h_{22} \cdot r_L}
\]

Using standard transistor \( h \) parameter nomenclature, its value for CE arrangement is

\[
A_I = \frac{h_{je}}{1 + h_{oe} \cdot r_L}
\]

The reader can readily write down the expressions for CB and CC arrangements.

\( iii \) Voltage gain. The general expression for voltage gain is

\[
A_V = \frac{-h_{21}}{Z_{in} \left( h_{22} + \frac{1}{r_L} \right)}
\]

Using standard transistor \( h \) parameter nomenclature, its value for CE arrangement is:

\[
A_V = \frac{-h_{je}}{Z_{in} \left( h_{oe} + \frac{1}{r_L} \right)}
\]

In the same way, expressions for voltage gain in CB and CC arrangement can be written.

\( iv \) Output impedance. The general expression for output impedance is

\[
Z_{out} = \frac{1}{h_{22} - \frac{h_{21} \cdot h_{12}}{h_{11}}}
\]

Using standard transistors \( h \) parameter nomenclature, its value for CE arrangement is:

\[
Z_{out} = \frac{1}{h_{oe} - \frac{h_{je} \cdot h_{re}}{h_{1e}}}
\]

The above expression for \( Z_{out} \) is for the transistor. If the transistor is connected in a circuit to form a single stage amplifier, then output impedance of the stage = \( Z_{out} \parallel r_L \) where \( r_L = R_C \parallel R_I \).
Example 24.6. Fig. 24.11 shows the transistor amplifier in CE arrangement. The h parameters of transistor are as under:

\[ h_{ie} = 1500 \, \Omega; \quad h_{fe} = 50; \quad h_{re} = 4 \times 10^{-4}; \quad h_{oc} = 5 \times 10^{-5} \]

Find (i) a.c. input impedance of the amplifier (ii) voltage gain and (iii) output impedance.

Solution. The a.c. load \( r_L \) seen by the transistor is equivalent of the parallel combination of \( R_C (= 10 \, \text{k}\Omega) \) and \( R_L (= 30 \, \text{k}\Omega) \) i.e.

\[
\frac{R_C R_L}{R_C + R_L} = \frac{10 \times 30}{10 + 30} = 7.5 \, \text{k}\Omega = 7500 \, \Omega
\]

\[ r_L + V_{cc} = 20 \, \text{V} \]

\[ \quad 80 \, \text{k}\Omega \quad 10 \, \text{k}\Omega \]

\[ 40 \, \text{k}\Omega \quad 10 \, \text{k}\Omega \]

\[ 30 \, \text{k}\Omega \quad V_{out} \]

\[ \text{Fig. 24.11} \]

(i) The input impedance looking into the base of transistor is given by:

\[
Z_{in} = h_{ie} - \frac{h_{oc} h_{fe}}{h_{oc} + \frac{1}{r_L}} = 1500 - \frac{4 \times 10^{-4} \times 50}{5 \times 10^{-5} + \frac{1}{7500}} = 1390 \, \Omega
\]

This is only the input impedance looking into the base of transistor. The a.c. input impedance of the entire stage will be \( Z_{in} \) in parallel with bias resistors i.e.

Input impedance of stage = \( 80 \times 10^3 \parallel 40 \times 10^3 \parallel 1390 = 1320 \, \Omega \)

(ii) Voltage gain, \( A_v = \frac{-h_{fe}}{Z_{in} \left( h_{oc} + \frac{1}{r_L} \right)} = \frac{-50}{1390 \left( 5 \times 10^{-5} + \frac{1}{7500} \right)} = -196 \)

The negative sign indicates phase reversal. The magnitude of gain is 196.

(iii) Output impedance of transistor is

\[
Z_{out} = \frac{1}{h_{oc} - \frac{h_{fe} h_{re}}{h_{ie}}}
\]

\[
= \frac{1}{5 \times 10^{-5} - \frac{50 \times 4 \times 10^{-4}}{1500}} = 27270 \, \Omega = 27.27 \, \text{k}\Omega
\]

\[ \therefore \text{Output impedance of the stage} = Z_{out} \parallel R_L \parallel R_C \]

\[ = 27.27 \, \text{k}\Omega \parallel 30 \, \text{k}\Omega \parallel 10 \, \text{k}\Omega = 5.88 \, \text{k}\Omega \]

**Approximate Hybrid Formulas for Transistor Amplifier**

The h-parameter formulas (CE configuration):
(i) Input impedance

Input impedance, \( Z_{\text{in}} = h_{\text{ie}} - \frac{h_{\text{re}} \cdot \frac{h_{\text{fe}}}{1 + \frac{1}{r_L}}} {h_{\text{oe}} + \frac{1}{r_L}} \)

In actual practice, the second term in this expression is very small as compared to the first term.

\[ Z_{\text{in}} = h_{\text{ie}} \quad \text{... approximate formula} \]

(ii) Current gain

Current gain, \( A_i = \frac{h_{\text{fe}}}{1 + h_{\text{oe}} \cdot r_L} \)

In actual practice, \( h_{\text{oe}} r_L \) is very small as compared to 1.

\[ A_i = h_{\text{fe}} \quad \text{... approximate formula} \]

(iii) Voltage gain

Voltage gain, \( A_v = -\frac{h_{\text{fe}}}{Z_{\text{in}} \left( \frac{1}{h_{\text{oe}} + \frac{1}{r_L}} \right)} \)

\[ = -\frac{h_{\text{fe}} \cdot r_L}{Z_{\text{in}} \left( h_{\text{oe}} r_L + 1 \right)} \]

Now approximate formula for \( Z_{\text{in}} \) is \( h_{\text{ie}} \). Also \( h_{\text{oe}} r_L \) is very small as compared to 1.

\[ A_v = -\frac{h_{\text{fe}} \cdot r_L}{h_{\text{ie}}} \quad \text{... approximate formula} \]

(iv) Output impedance

Output impedance of transistor, \( Z_{\text{out}} = h_{\text{oe}} - \frac{h_{\text{fe}} \cdot h_{\text{re}}}{h_{\text{ie}}} \)

The second term in the denominator is very small as compared to \( h_{\text{oe}} \).

\[ Z_{\text{out}} = \frac{1}{h_{\text{oe}}} \quad \text{... approximate formula} \]

The output impedance of transistor amplifier

\[ = Z_{\text{out}} \parallel r_L \quad \text{where } r_L = R_C \parallel R_L \]

If the amplifier is unloaded (i.e. \( R_L = \infty \)), \( r_L = R_C \).
**Example 24.9.** For the circuit shown in Fig. 24.13, use approximate hybrid formulas to determine (i) the input impedance (ii) voltage gain. The $h$ parameters of the transistor are $h_{ie} = 1.94 \, k\Omega$ and $h_{fe} = 71$.

![Fig. 24.13](image)

**Solution.**

(i) a.c. collector load, $r_L = R_C || R_L = 12 \, k\Omega || 15 \, k\Omega = 6.67 \, k\Omega$

(ii) Transistor input impedance is

$$Z_{in \ (base)} = h_{ie} = 1.94 \, k\Omega$$

Circuit input impedance

$$Z_{in \ (base)} || R_1 || R_2 = 1.94 \, k\Omega || 50 \, k\Omega || 5 \, k\Omega = 1.35 \, k\Omega$$

(ii) Voltage gain, $A_v = \frac{h_{fe} \cdot r_L}{h_{ie}} = \frac{71 \times 6.67 \, k\Omega}{1.94 \, k\Omega} = 244$